from the edge, provide evidence of the movement of a local shock wave through the free shear layer. The velocity measurements now available from these observations have an uncertainty of at least 20%; however, they are quite consistent with velocity measurements from the shocklets' Mach angles.

From these results, we can speculate that a functional relationship exists between the strength of a turbulent shocklet and Reynolds number. We know that, in the early stages of turbulence, there is a confirmed relationship between turbulent intensity and Reynolds number;[9] our observations may be hostage to this relationship. However, the relationship between turbulent intensity and Reynolds number has a characteristic frequency as its control parameter. Therefore, an adjustment in the characteristic frequency can change the nature of this relationship and, in turn, change the strength of the shocklets. The Reynolds number is also altered, of course, by changes in characteristic length and in viscosity and local velocity; this means that direct variation in Reynolds numbers, even at fixed flow velocity, can cause a direct variation in the strength of the shocklets. Since there is an apparent influence of Reynolds number on the strength of the shocklets, the options cited allow us to anticipate the control of turbulent intensity as an avenue for the control of the supersonic mixing.

Conclusions

The systems of internal shock waves produced in turbulent free shear layers are correlated with Reynolds number. This correlation implies a direct connection with turbulent intensity. Through this connection and the adjustability of the Reynolds number, some opportunities may be found for the control of turbulent shocklets and thereby the control of their influence on supersonic mixing.

Acknowledgment

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Optimum Synthesis of Polymer Matrix Composites for Improved Internal Material Damping Characteristics

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Introduction

THE present paper examines the optimum internal material damping characteristics of short-fiber, polymer matrix composites. Internal damping depends the properties of the fiber and matrix materials and on the geometrical layout of the composite. Damping properties of continuous and short-fiber composites have been studied and documented in the literature.[1] Glass- and graphite-reinforced polymer matrix composites exhibit anisotropic, linear viscoelastic behavior, and the principal mechanism of damping in such composites is considered to be the viscoelastic energy dissipation in the matrix material. Stress concentration effects in discontinuous fiber composites facilitate the transfer and dissipation of energy in the viscoelastic polymer matrix and, therefore, yield a higher level of internal damping.

The elastic-viscoelastic correspondence principle has been used in conjunction with a force-balance approach to obtain analytical estimates of internal damping in short-fiber composites.[2] This approach essentially develops relations for the loss and storage modulii of the composite in terms of the fiber aspect ratio, loading angle, stiffness of fiber and matrix materials, the fiber volume fraction, and the damping properties of the fiber and matrix materials. The present paper proposes a design synthesis procedure based on formal multidimensional optimization in which the extensional loss factor of a representative volume element is maximized, subject to constraints on the element mass and stiffness characteristics. Two distinct analysis models, one that uses Cox's shear-lag theory,[3] and another based on an advanced shear-lag theory that allows for the matrix to partially sustain extensional loads,[4] are used in the present work. These analytical models and their adaptation in optimum synthesis are discussed next.

Analysis and Optimum Synthesis Problem

A representative volume element, shown in Fig. 1, describes the geometrical relationship between the fiber and matrix materials. Short fibers of length s and diameter d are embedded in a matrix element of length $s+p$ and diameter D. The ratio s/d is referred to as the fiber aspect ratio and p/s denotes the discontinuity ratio. The extensional modulus in the x direction is written as a complex quantity

$$E_x^* = E_x' + iE_x'' \tag{1}$$

where E_x' and E_x'' are the storage and loss modulii, respectively; the ratio $\eta_x = E_x''/E_x'$ is the loss factor and a measure of the internal damping. The expression for E_x^* is obtained in terms of longitudinal modulus E_L^*, transverse modulus E_T^*, shear modulus G_{LT}^*, and Poisson's ratio ν_{LT}, where the transverse and shear modulii are obtained by the use of the Halpin-Tsai equations and the rule of mixtures. The longitudinal modulus is derived from the force equilibrium of the compos-

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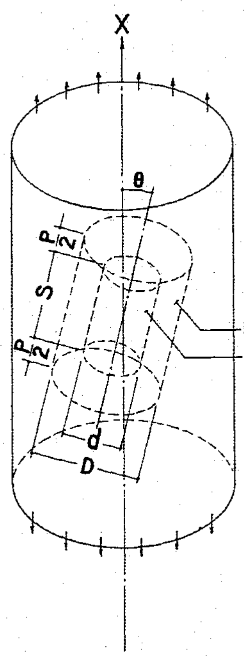

Fig. 1 A representative volume element for off-axis loading.

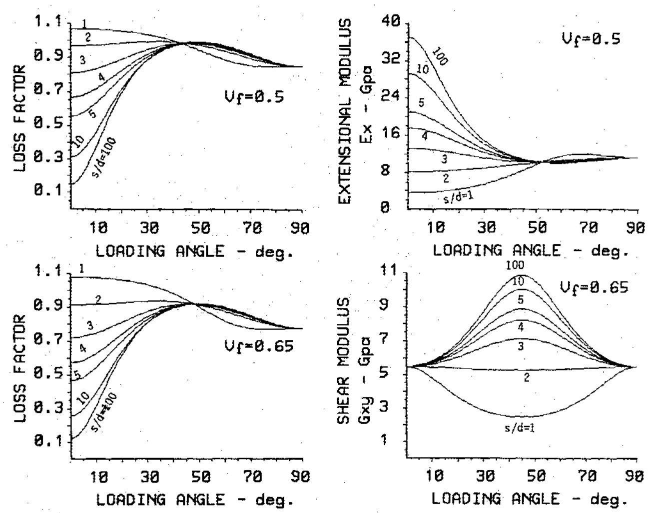

Fig. 2 Parametric variations of loss factor, extensional modulus, and shear modulus with fiber aspect ratio and loading angle. $V_f = 0.5$ and 0.65 are for square packing and hexagonal packing geometry, respectively.

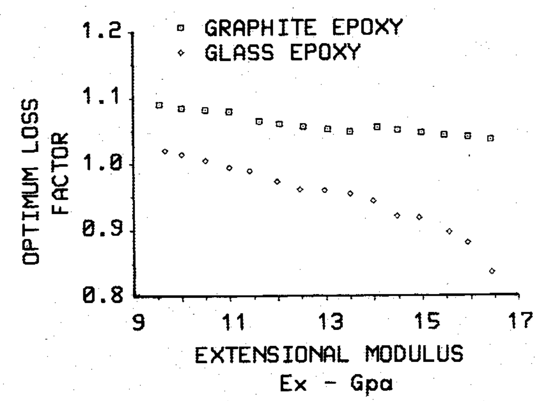

Fig. 3 Variations in optimum loss factor for different levels of extensional modulus for the composite specimen.

ite specimen and needs the average stress distribution in the fiber. If Cox's shear-lag analysis is used to determine the rate of transfer of load from the fiber to the matrix material, the expression for E_x^* can be written as

$$1/E_x^* = 1/(E_x' + iE_x'') = \cos^4\theta/E_L^* + \sin^4\theta/E_T^* + (1/G_{LT}^* - 2\nu_{LT}/E_L^*)\sin^2\theta\cos^2\theta \qquad (2)$$

Here, the longitudinal, transverse, and shear modulii depend on the component material moduli that are complex quantities and the volume fraction of the composite specimen that is dependent on the packing geometry. An alternative model that accounts for the tensile load-carrying capacity of the matrix material and influences the load distribution between the fiber and matrix material was also used in this study. The additional assumption of this model was to consider the matrix material between the fiber ends as an extension of the fiber with a different tensile modulus. The expression for the loss factor is then obtained as a function of the ratio $k = E_m/E_f$, which for $k = 0$, reduces to the Cox's theory.

In the present work, the analysis for damping was reformulated as a formal multidimensional optimization problem, with the following mathematical statement. Maximize

$$\eta_x(V) \qquad (3)$$

subject to

$$gj(V) < 0, \qquad j = 1, 2, \ldots, m \qquad (4)$$

and

$$V_i^1 < v_i < V_i^u, \qquad i = 1, 2, \ldots, n \qquad (5)$$

where

$$V = \{E_f^*, E_m^*, G_m^*, p, s, d, D, \theta\} \qquad (6)$$

Here, V, is the vector of design parameters and variables and is defined as above. The side constraints V_i^1 and V_i^u were formulated for nominal values of lower and upper bounds on the geometric variables. The bounds selected for the present implementation are $1 \le s/d \le 1000$, $d \le D - 0.001$, $2 \le P/d \le 5$, $0.05 \le p/s$, $0 \le \theta \le \pi/2$, and the fiber volume fraction is allowed to range between 0.5 and $\pi/4$ for square packing, and between 0.5 and $\pi/3.464$ for hexagonal packing geometry. In addition to the side constraints just described, additional inequality constraints were imposed on the mass of the representative volume element and on its extensional and shear stiffness. The optimization problem described was solved by using a nonlinear-programming-based, feasible, usable search-direction algorithm.

Discussion of Results

At the very outset, a systematic parametric study of glass-epoxy and graphite-epoxy composites was conducted to examine the variation of damping with volume fraction, fiber aspect ratio, and the loading angle. Similar variations were also obtained for the extensional and shear moduli, and the results obtained for the glass-epoxy material are shown in Fig. 2. It is very obvious that a comprehensive parametric study of all possible combinations would be extremely cumbersome and optimum design synthesis methods present an attractive alternative approach to studying the problem.

The multivariable optimization problem was solved for optimum damping, both with and without constraints on the extensional stiffness. Optimum damping was obtained for both isotropic glass and orthotropic graphite fibers as a function of the bounds on extensional stiffness and is shown in Fig. 3. Proper scaling of design variables and convergence to an optimal design from several starting estimates of design variables was necessary to eliminate convergence to a local optimum. Estimates of damping based on the modified shear-lag analysis were obtained in a similar manner. For the glass- and graphite-reinforced composites, the ratio E_m/E_f was small, and the results were not significantly different from those obtained in Cox's shear-lag theory.

Some basic conclusions can be drawn from the results obtained in this study. Similar numerical trends were obtained for the isotropic glass and orthotropic graphite fibers. The parametric variation in damping with the loading angle and fiber aspect ratio indicates that the latter is more critical to damping at lower values of loading angles. Furthermore, an increase in volume fraction results in lower damping because of a decrease in the amount of polymer matrix material. Requirements of a higher extensional stiffness resulted in lower values of damping. This is not unusual as stiffness is governed by the fiber content, and its increase results in a corresponding decrease in the matrix material. Although the packing geometry introduced no significant difference in the magnitude of the damping factor, the upper bound on the volume fraction of the hexagonal geometry is about 15% higher. This allows for an increased stiffness of the specimen.

Since the matrix is assumed to sustain partially axial loads in the modified shear-lag model, one would expect a reduction in the interfacial shear stress in short-fiber composites and a correspondingly lower value of damping. The very low values of E_m/E_f obtained for the materials under study yielded estimates of damping close to those obtained from Cox's model. The difference would be more significant in metal matrix materials, where the ratio E_m/E_f would have a larger value. The present work is being extended to study this aspect and to also examine other fiber arrangements by the modified shear-lag approach.

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